Symmetries of the plane

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An implementation of all 17 plane symmetries in TikZ. With the orbifold signature these are the following:

\[
\begin{array}{cccc}
*632 & *442 & *333 & *2222 & ** \\
2*22 & *× & \ \\
4*2 & 3*3 & 22* & \ \\
22× & ×× & \ \\
632 & 442 & 333 & 2222 & ◦
\end{array}
\]

For their meaning and more pretty pictures see “The Symmetries of Things” by John H. Conway, Heidi Burgiel and Chaim Goodman-Strauss.

A brief comment about the orbifold signature: A number \( n \) before * represent a gyration point of order \( n \), thus in 632 we have three effectively distinct points with respective orders 6, 3 and 2. Each * represent a kaleidoscope, a kaleidoscope which is not followed by a number is a mirror, a following number \( n \) represent a point where \( n \) mirror lines cross. Thus, in *333 we have a kaleidoscope with 3 different points, in each one, there are crossing 3 mirrors. In 4*2 we have a gyration point of order 4, the four neighbour point are actually the same point, a point where 2 mirrors cross. The × is called a miracle and represent a reflection which cannot be explained by a mirror line. Finally, the ◦ is called a wonder and represents translations which are not explained by any previous symbol.