ZASSENHAUS' LEMMA

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Zassenhaus' Lemma states that:

Theorem (Zassenhaus' Lemma)**.** *Let* $A \trianglelefteq A^*$ *and* $B \trianglelefteq B^*$ *be subgroups of* G*. Then,*

$$A(A^* \cap B) \trianglelefteq A(A^* \cap B^*) \tag{1}$$

$$B(B^* \cap A) \trianglelefteq B(B^* \cap A^*) \tag{2}$$

and we have an isomorphism:

$$\frac{A(A^* \cap B^*)}{A(A^* \cap B)} \simeq \frac{B(B^* \cap A^*)}{B(B^* \cap A)} \tag{3}$$

The picture below represents the butterfly that goes with its proof.

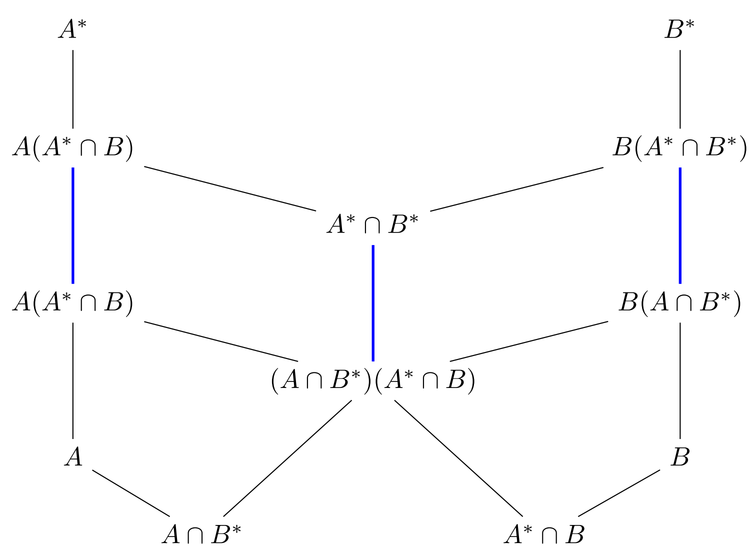

Figure 1. The Butterfly

In the figure, the quotients given by the blue line are isomorphic to each other thus proving the Zassenhaus' Lemma. Further, the black line indicates that the group that lies below is normal in the groups connected to it and lies above it in the plane of the figure.